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Effects of flow oblique angle on three-dimensional steady streaming at low *Keulegan-Carpenter* number

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Abstract

This paper concerns the effects of flow oblique angle on steady streaming around a circular cylinder in oscillatory flow. Three-dimensional (3D) direct numerical simulations are conducted at Keulegan-Carpenter (*KC*) number of 2 and frequency parameter (β) of 400. The oblique angle is defined as the angle between the incoming flow direction and the direction perpendicular to the cylinder axis. The calculated steady streaming field is studied by way of vorticity cores and streamlines. For the perpendicular incoming flow under the governing parameters specified, mutually interacting vortices along the cylinder span are observed when looking at instantaneous flow structures. The steady streaming results show well-defined vortices distribution as the three-dimensional instability is mostly smoothed out during the time-averaging. For an oblique incoming flow, the existence of an axial flow component changes the features of steady streaming observed for the perpendicular case. The vortical tubes are shortened and obliquely oriented, and the vorticity field near the cylinder surface suffers a decrease in strength. Both the instantaneous flow development and the time-averaged steady streaming show that as the magnitude of the axial flow component increases, the resulted steady streaming field experiences a gradually reduction of three-dimensionality and eventually switches to two-dimensional.

Introduction

Extensive research has been conducted on oscillatory flow around a circular cylinder due to its prevalent engineering applications. Previous studies have been concerned with both instantaneous and time-averaged flow characteristics when dealing with oscillatory flow around a circular cylinder. The interaction between the fluid particles and the cylinder in the flow field induces a non-zero period-averaged flow around the cylinder, which has significant effects on mass/momentum/energy transfer around a cylinder in oscillatory flow. The resulted non-zero mean flow field is referred to as steady streaming in relevant studies ([11,12]). The origin of the steady streaming is attributed to either the non-conservative oscillating body force or the action of the Reynolds stress in the thin Stokes layer near the solid boundary where a no-slip boundary condition applies ([8]).

It should be mentioned here that the two governing parameters for an oscillatory flow, the Keulegan-Carpenter number (*KC*) and the frequency number (β), can be defined as below:

$$KC = \frac{U_{\mathrm{M}}T}{D} \quad \text{and} \quad \beta = \frac{Re}{KC} = \frac{D^2}{\nu T} \tag{1}$$

where U_{M} and T are the magnitude and period of flow oscillation respectively, D is the diameter of the cylinder and ν is the kinematic viscosity. Previous theoretical developments on steady streaming were achieved mostly under the flow condition at $KC \ll 1$ and large β values ([12,13]), when the whole flow field remains two-dimensional. For moderate governing parameters, experiments have been reported on the study of steady streaming phenomenon ([5,7,14]). When *KC* is large enough, a transition from two-dimensional to three-dimensional in the flow field is likely to occur, as first reported by Honji [4]. The transition takes the form of several mushroom-shaped vortex pairs distributed along the cylinder span. The instantaneous flow characteristics of Honji structures were studied previously through both physical experiments and numerical simulations ([3,9,10,15,16,17]). The time-averaged flow field for Honji instability regime was investigated through the examination of the calculated steady streaming by An et al. [2]. Steady streaming features of flow at $KC = 2$ is characterized by two-dimensional flow ($\beta = 100$), stable Honji vortices ($\beta = 200$), unstable Honji vortices ($\beta = 400$) and turbulent flow ($\beta = 600$) regimes.

This paper extends the work of An et al. [2,3] to address the effects of flow oblique angle on the resulted steady streaming features under the unstable Honji instability regime. The governing parameters chosen for the present study are $KC = 2$ and $\beta = 400$. Comparison is made between the steady streaming obtained from the flow field under normal incidence case and the oblique inflow cases.

Methodologies

Governing Equations

The oscillatory flow is predicted by solving the incompressible Navier-Stokes and continuity equations when an oblique oscillatory flow impinges on a circular cylinder. These equations, following the restrictions of incompressibility, can be normalised in the Cartesian coordinates as below

Navier-Stokes equation

$$\frac{1}{KC}\frac{\partial \mathrm{u_i}}{\partial t} + u_j\frac{\partial \mathrm{u_i}}{\partial x_j} = -\frac{\partial \mathrm{p}}{\partial x_i} + \frac{1}{\beta KC}\frac{\partial^2 \mathrm{u_i}}{\partial {x_j}^2} \tag{2}$$

and continuity equation

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{3}$$

where $i, j = x, y$ and z; the dimensionless quantities in the above equations are non-dimensionalised as follows: $x_i = x_i'/D$; $u_i = u_i'/U_{Mx}$; $p = p'/\rho U_{Mx}^2$ and $t = t'/T$, where x_i' is the coordinate component, u_i' is the velocity component, p' is pressure, t' is time, ρ is fluid density. The governing parameters, KC and β, are defined as in (1) but using the flow component perpendicular to the cylinder axis (U_{Mx}).

Numerical Method

As a succession of [2,3], the current study applies the same computational model. The governing equations given above are solved using a Petrov-Galerkin finite element method. In order to eliminate all possible influencing factors other than the inflow angle, same computational domain and discretization method as [2,3] are used in the present study. In the computational domain with a dimension of $40D\times20D\times4D$, the total number of elements is 448,950 with 96 nodes distributed around the perimeter of the cylinder and 72 nodes in the axial direction. For more details about the numerical scheme and the meshing method, interested readers are directed to [3].

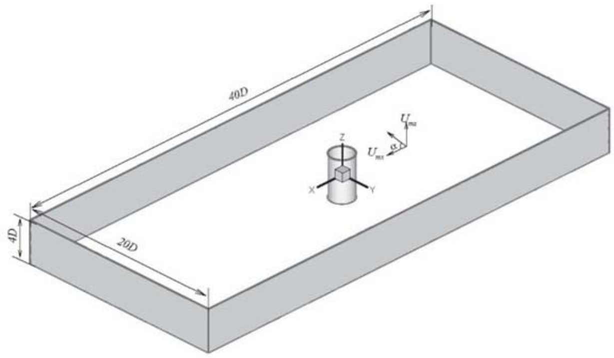

Figure 1. Sketch of computational domain.

As shown in figure 1, the cylinder, with its axis in the z-direction, is positioned in the centre of the x-y plane with the co-ordinates oriented so that far from the cylinder the flow oscillates in the x-z plane. The oblique inflow angle, which is denoted as α, is defined as the angle between the approaching flow and the normal direction of the cylinder axis. It follows this definition that larger values of α leads to an increased value of the axial flow component. This leads to the inlet and outlet boundary conditions given as below

$$(u_x, u_y, u_z) = (1, 0, \tan\alpha)\sin 2\pi t \tag{4}$$

and the pressure at these two boundaries is given by analytical solution for undisturbed flow. Symmetric boundary condition is enforced on the two bounding surfaces parallel to the x-z plane and no-slip boundary condition is imposed at the cylinder. For the cylinder to be representing that of infinite length, a pressure condition is given at two ends of the cylinder by

$$\frac{\partial \mathrm{p}}{\partial z} = -\frac{2\pi}{KC}\tan\alpha\cos 2\pi t \tag{5}$$

The calculation is started with a state of rest, i.e. the initial velocity in the whole domain is zero. 200 flow periods are simulated for each case to allow the flow features to fully develop.

The steady streaming has been calculated taking the time-averaged velocity field within one period, according to its general definition. The steady streaming velocity in this study is computed as below

$$\overline{u_i} = \int_{199T}^{200T} u_i / T dt \quad i = x, y, z \tag{6}.$$

Results & Discussion

Flow development

The instantaneous flow development process is illustrated by the temporal and spatial variation of the relative magnitude of the axial flow component u_z/U_{mx} along a probing line of $(x, y) = (0, 0.51)$ near the cylinder surface, as shown in figure 2. The three-dimensionality of the flow field is reflected by the variation of grey scales in the figure. Each strip represents one Honji vortex as explained by An et al [2]. When the flow approaches the cylinder perpendicularly (figure 2 (a)), the vortices along the cylinder span interact with each other through a process of amalgamation and splitting, and the flow belongs to the 'unstable Honji regime' ([2,3,15]). Figure 2 (b) shows the developing process of three-dimensionality of the case of α = 20°. It is observed that the interaction frequency is much lower than that shown in figure 2 (a). The irregularity of the vortical developing process also reduced in this case and some clear pattern can be seen. When α is increased to 30°, six parallel strips are formed after the flow steps into three-dimensional as shown in figure 2 (c), indicating full suppression of interactions between vortices. No strip is observed in figure 2 (d), where α = 40°, demonstrating a purely two-dimensional flow with no axial-variation of the vortical structures under current flow condition. This suppression effect of the axial flow component induced by an oblique incoming flow on the three-dimensional of oscillatory flow around a circular cylinder was reported in more detail by Yang et al. [15].

Further increase of α will lead to an axial-flow-dominated condition and different flow instability regimes may occur. However that topic is outside the scope of the present work.

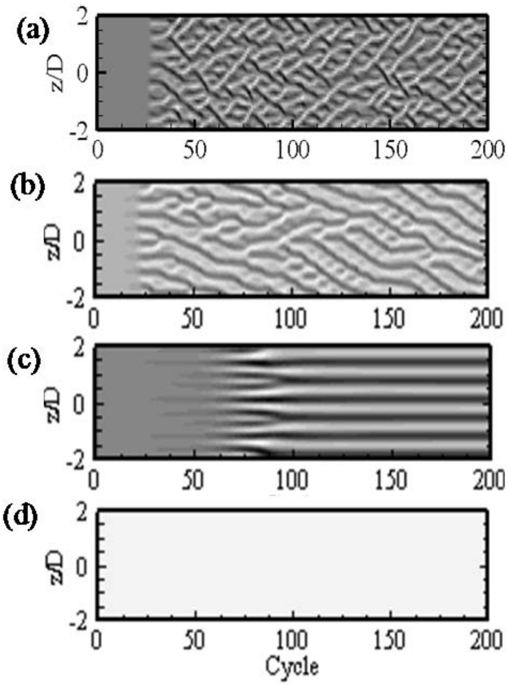

Figure 2. Temporal and special variations of relative axial flow velocity component u_z/U_{mx}. Flow development for KC = 2, β = 400 at (a) α = 0°; (b) α = 20°; (c) α = 30°; (d) α = 40°.

Steady streaming

The resulted three-dimensional steady streaming is visualised by way of streamlines and iso-surface of an eigenvalue (λ_2). The iso-surface of the eigenvalue λ_2 represents the locations of vortex cores, as proposed by Jeong & Hussain [6]. The steady streaming for a perpendicular approaching flow ($\alpha = 0°$) is given in figure 3. The cylinder surface is covered by six pairs of rib-like vortex tubes, with an obvious difference in the strength of the vortices within each pair. The rib-like vorticity tubes are perpendicular to the cylinder axis, as can also be reflected by the streamline demonstration given in figure 3 (b). As the approaching flow reaches the cylinder surface, the streamline goes up as it rotates, forming the vortices on the cylinder surface (figure 3 (c)).

The main features of steady streaming at $\alpha = 10°$ (not shown here) is very similar to that observed in figure 3 due to the small inclined angle of incoming flow. No more discussion is given on that case.

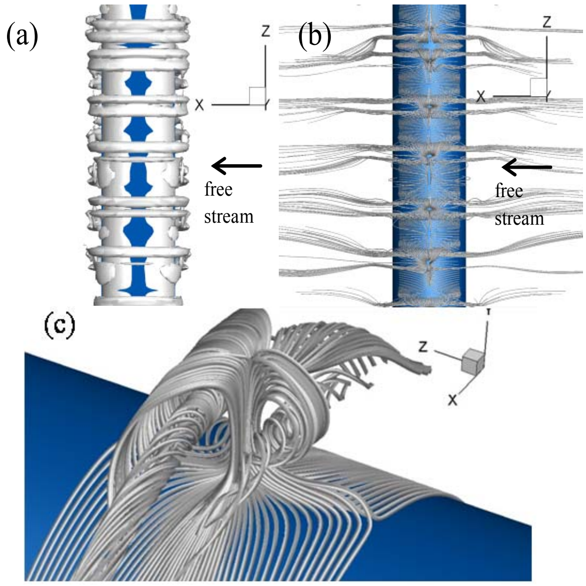

Figure 3. Steady streaming of case of $KC = 2$, $\beta = 400$ and $\alpha = 0°$. (a) eigenvalue λ_2 = -0.5; (b) streamline; (c) focus of the streamline for one pair of vortical structure.

With the increase of α value, obvious change in the steady streaming structures happens. The steady streaming of $\alpha = 20°$ is shown in figure 4, with figure 4 (a) the iso-surface of eigenvalue λ_2 = -0.5. Though strong irregularity of the three-dimensional vortical structures still exists, the rib-like vortical tubes are much shorter than that shown in figure 3 (a). The axial flow also causes the rib-like tubes to oblique to the x-axis. Closer examination for each pair of the vortical tubes shows that the two constituting parts no longer sit side-by-side (symmetric with regard to z-axis) as is the case for $\alpha = 0°$, but dislocate in a staggered fashion with unbalanced strength. The streamlines for this case (figure 4 (b) (c)) show that the far field flow approaches the cylinder at an angle of about 20°, but passes the cylinder at a slightly smaller oblique angle (this feature is more apparent for larger α values to be discussed later). The vortices on the cylinder surface are less focused, hence the rotation seems to be looser (figure 4 (c)).

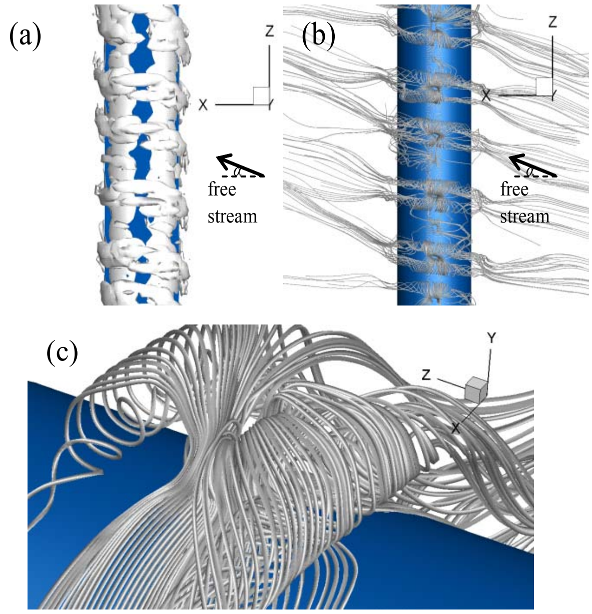

Figure 4. Steady streaming of case of $KC = 2$, $\beta = 400$ and $\alpha = 20°$. (a) eigenvalue λ_2 = -0.5; (b) streamline; (c) focus of the streamline for one pair of vortical structure.

Further increase the oblique angle to 30° suppresses the formation of the rib-like vortex pairs and the interaction between two adjacent vortical structures. As seen in figure 5 (a), the iso-surface of eigenvalue λ_2 = -0.5 on the positive y side of the cylinder surface is divided into two isolated groups, covering yet less circumference of the cylinder than that of $\alpha = 20°$, and each group is consisted of six evenly distributed vortices with similar strength. The streamlines given in figure 5 (b) (c) confirm that the rib-like vortex pairs as those observed in figure 3 and figure 4 cease to form, as the streamlines do not swirl around cylinder surface before flowing out along the incoming flow direction (figure 5 (b) (c)).

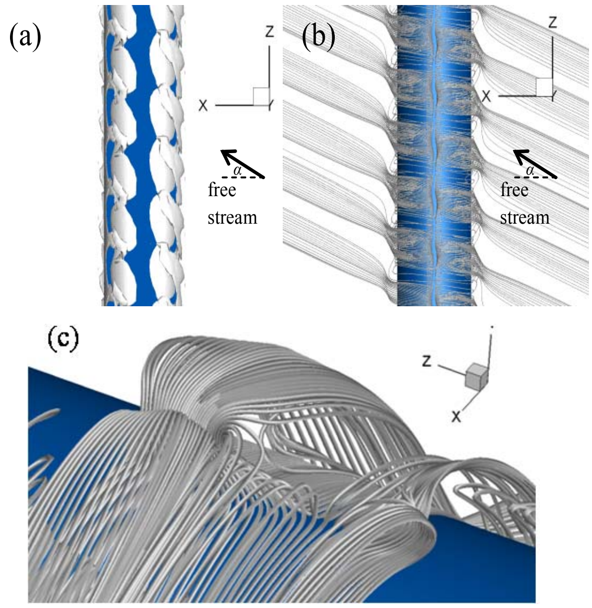

Figure 5. Steady streaming of case of $KC = 2$, $\beta = 400$ and $\alpha = 30°$. (a) eigenvalue λ_2 = -0.5; (b) streamline; (c) focus of the streamline for one pair of vortical structure.

When α is increased to 40°, the flow remains two-dimensional. As can be seen in figure 6, no apparent variation in the axial direction is observed either for the iso-surface of the eigenvalue or the streamlines. A zoom-in view of the streamline in figure 6 (c) shows that, as the flow approaches the cylinder under current α value, the cylinder surface is simply attached by two-dimensional vortices due to no-slip boundary condition enforced on its surface. The two groups of vortices are not symmetric with respect to z-axis, but there's again a shift of relative location along the cylinder axis between the two counterparts, caused by the angle of attack.

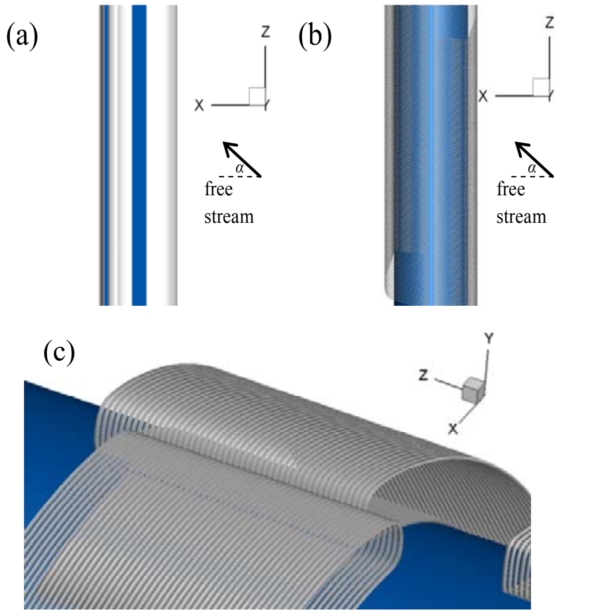

Figure 6. Steady streaming of case of $KC = 2$, $\beta = 400$ and $\alpha = 40°$. (a) eigenvalue $\lambda_2 = -0.5$; (b) streamline; (c) zoom-in view of streamlines.

Conclusions

The paper serves as a further work to that of An et al. [2,3] in discussing the effects of an axial flow on the alternation of the steady streaming features through a three-dimensional DNS study on oscillatory flow around a circular cylinder at $KC = 2$ and $\beta = 400$. The steady streaming field is calculated by time-averaging of the flow velocity field within one flow oscillation period.

Perpendicular approaching flow under the specified governing parameters falls into the 'unstable Honji' regime, when the Honji instability is observed and featured by strong unstable Honji vortices interacting with each other along the cylinder span. However, the time-averaged steady streaming field with most three-dimensional disturbances smoothed out by the calculating process gives us the distinctive rib-like vortex pairs distributed along the cylinder span for $\alpha = 0°$. For $\alpha = 10°$ and $20°$, the rib-like vortical tubes orient obliquely to the cylinder axis with an inclination angle smaller than α. The suppression effect of the axial flow component as reflected by the instantaneous flow development also applies to the time-averaged steady streaming field. Weak three-dimensionality still exist at $\alpha = 30°$ but is completely suppressed by $\alpha = 40°$, when the flow remains two-dimensional. Furthermore, examination of the streamlines for different α values demonstrates that as α increases, the vortices are less focused and weaker, leading to the breakdown of the three-dimensional instability. It is also found that as the free stream approaching the cylinder at an oblique angle of α gets near the cylinder, it adjusts its direction slightly and finally passes the cylinder more perpendicular to the cylinder axis.

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